Factoring

Reporting Category	Expressions and Operations
Topic	Factoring polynomials
Primary SOL	AII.1d The student, given rational, radical, or polynomial expressions, will factor polynomials completely.
Related SOL	AII.8

Materials

- Graphing calculators
- Two attached handouts

Vocabulary

factor, prime, composite, greatest common factor, binomial, trinomial (earlier grades)

sum and difference of two cubes, perfect square trinomial (AII.1d)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Review factoring, defining *factor* as both a noun and a verb. Show different ways to factor a composite number: e.g., 12 can be factored as 4 · 3, 6 · 2, or 2 · 2 · 3.
2. Review multiplication of polynomial expressions. Focus on distributing a monomial and on multiplying two binomials. Stress that when multiplying two binomials, students should pay particular attention to the sum of the like terms that are combined.
3. Show examples of factoring out a greatest common factor.
4. Have students factor several trinomials in the form $ax^2 + bx + c$, where $a = 1$. Remind them that the focus is identifying two numbers with a given product, c, and a given sum, b.
5. Direct students to factor $2c^2 + 6c - 20$. Some students will likely be confused. Stress the fact that the first step of factoring completely is to factor out the greatest common factor, if all terms have a common factor. Proceed to factoring the polynomial completely.
6. Have students factor several polynomial expressions, some with a common factor only, some trinomials in which $a = 1$, and some trinomials in which a is a common factor of all three terms.
7. Show additional examples of multiplying binomials that have leading coefficients other than one. These examples should remain on the board to reference during step #8.
8. Demonstrate factoring trinomials in which $a > 1$. Include examples of prime expressions.
9. Through examples of multiplication of binomials, lead students to discover the following factor patterns:
   - Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$
   - Perfect square trinomial: $a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
   - Sum of two cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
   - Difference of two cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

10. Distribute copies of the attached Factor Patterns handout, and have students complete it in small groups. Direct them to identify common factors and special patterns before factoring.
11. Distribute copies of the attached Factor, Factor, Factor! handout, and have students work in pairs to complete it. Then, have pairs share their steps to factoring the problems.

Assessment

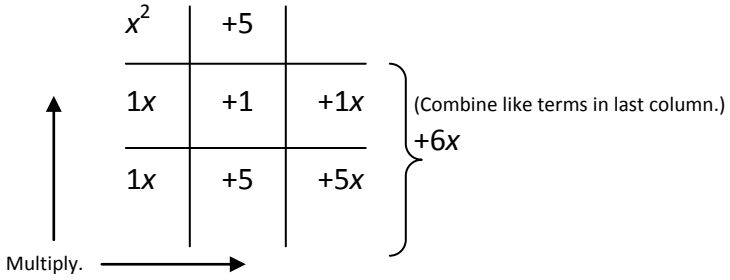
- **Questions**
  - When can the sum of two squares be factored?
  - What are the four special factor patterns?
- **Journal/Writing Prompts**
  - Explain why we do not consider $w^2 + 25$ a perfect square binomial.
  - Write a song to help you remember the steps taken to completely factor a polynomial expression.

Extensions and Connections (for all students)

- Have students graph the following functions on their calculators, one at a time:
  $y = x^2 - 2x - 15$ $y = x^2 + 6x + 9$ $y = x^2 - 4$ $y = 4x^2 + 12x + 5$
  Then, have them factor the polynomial expressions. Finally, have them discuss in pairs the connections that can be made between factors and x-intercepts.
- Give students several expressions in the form $ax^2 + bx + c$, and have them evaluate $b^2 - 4ac$.

Strategies for Differentiation

- Use algebra tiles to model factoring trinomials and the difference of two squares.
- Create a matching game that uses cards showing expressions in expanded form and corresponding cards showing the expressions in factored form.
- Have students use graphing calculators to find factors of a number. For example, to find the factors of 36, have them graph $y = \frac{36}{x}$ and use the table to find integral (x,y) pairs.
- Use a Tic-Tac-Toe grid to illustrate factoring a trinomial. For example, the grid below illustrates a method for factoring $x^2 + 6x + 5$.



- Have students use an acronym to remember the sign of the factored form of the sum or difference of two cubes.
- Use either of the handouts to create a game, such as a Tic-Tac-Toe game or a matching game.

Factor Patterns

Factor the following expressions completely.

1. $y^2 - 121$

2. $4a^2 - 49$

3. $81x^2 - 25y^2$

4. $200 - 2w^2$

5. $m^2 - 10m + 25$

6. $4f^2 + 12f + 9$

7. $49x^2 - 28xy + 4y^2$

8. $3p^2 - 30p + 75$

9. $c^3 - 8$

10. $2y^3 + 128$

11. $5 - 5n^3$

12. $4u^3 - 108w^{12}$

Factor, Factor, Factor!

Factor the following expressions completely.

1. $m^2 + 2m - 15$

2. $x^2 - 16$

3. $9a^2 + 6a - 8$

4. $v^3 + 1$

5. $4w^2 + 12w + 9$

6. $k^2 + 2km + m^2$

7. $128 - 2x^2$

8. $3q^2 + 3q + 12$

9. $8h^2 - 8h - 6$

10. $9r^3 - 27$

11. $225n^4 - p^6$

12. $16c^2 + 40cd + 25d^2$

13. $4y^2 + 4$

14. $a^6 - b^6$